- Are some players much better at these skills than others? Does this make the game feel unfair?
- Can players improve their skills with practice, leading to a feeling of mastery?
- Does this game demand the right level of skill?

Exercising skills can be a joyful thing—it is one of the reasons that people love games. Of course, it is only joyful if the skills are interesting and rewarding and if the challenge level strikes that ideal balance between "too easy" and "too hard." Even dull skills (such as pushing buttons) can be made more interesting by dressing them up as virtual skills and providing the right level of challenge. Use this lens as a window into the experience the player is having. Because skills do so much to define experience, the Lens of Skill works quite well in conjunction with Lens #2, *Essential Experience*.

Mechanic 7: Chance

Our seventh and final game mechanic is **chance**. We deal with it last because it concerns interactions between all of the other six mechanics: space, time, objects, actions, rules, and skills.

Chance is an essential part of a fun game because chance means uncertainty, and uncertainty means surprises. And as we have discussed earlier, surprises are an important source of human pleasure and the secret ingredient of fun.

We must now proceed with caution. You can never take chance for granted, for it is very tricky—the math can be difficult, and our intuitions about it are often wrong. But a good game designer must become the master of chance and probability, sculpting it to his will, to create an experience that is always full of challenging decisions and interesting surprises. The challenges of understanding chance are well illustrated by a story about the invention of the mathematics of probability—invented, not surprisingly, for the express purpose of game design.

Invention of Probability

Il est tres bon ésprit, mais quel dommage, il n'est pas geometre.

(He's a nice guy, but unfortunately, no mathematician.)

—Pascal to Fermat regarding the Chevalier de Méré

It was the year 1654, and French nobleman Antoine Gombaud, the Chevalier de Méré (pronounced "Shevulyay duh Mayray"), had a problem. He was an avid gambler and had been playing a game where he would bet that if he rolled a single die four times, at least one time it would come up as a six. He had made some good money from this game, but his friends got tired of losing and refused to play it with him any further. Trying to find a new way to fleece his friends, he invented a new game that he believed had the same odds as the last one. In his new game, he would bet that if he rolled a pair of dice twenty-four times, a twelve would come up at least once. His friends were wary at first but soon grew to like his new game, because the Chevalier started losing money fast! He was confused, because by his math, both games had the same odds. Chevalier's reasoning was as follows:

First Game: In four rolls of a single die, the Chevalier wins if at least one six comes up.

The Chevalier reasoned that the chance of a single die coming up 6 was 1/6, and therefore rolling a die four times should mean the chance of winning was

$$4 \times (1/6) = 4/6 = 66\%, \text{ which explained why he tended to win.}$$

Second Game: In twenty-four rolls of a pair of dice, the Chevalier wins if at least one 12 comes up.

The Chevalier determined that the chance of getting a 12 (double sixes) on a pair of dice was 1/36. He reasoned, then, that rolling the dice 24 times meant the odds should be

$$24 \times (1/36) = 24/36 = 2/3 = 66\%. \text{ The same odds as the last game!}$$

Confused and losing money, he wrote a letter to mathematician Blaise Pascal, asking for advice. Pascal found the problem intriguing—there was no established mathematics to answer these questions. Pascal then wrote to his father's friend, Pierre de Fermat, for help. Pascal and Fermat began a lengthy correspondence about this and similar problems and, in discovering methods of solving them, established probability theory as a new branch of mathematics.

What are the real odds of Chevalier's games? To understand that, we have to get into some math—don't fret, it's easy math that anyone can do. Fully covering the mathematics of probability is not necessary for game design (and beyond the scope of this book), but knowing some of the basics can be quite handy. If you are a math genius, you can skip this section, or at least read it smugly. For the rest of us, I present the following:

Ten Rules of Probability Every Game Designer Should Know

Rule #1: Fractions Are Decimals Are Percents

If you are one of those people who has always had a hard time with fractions and percents, it's time to face up and deal with them, because they are the language of probability. Don't stress—you can always use a calculator—no one is looking.

The thing you have to come to grips with is that fractions, decimals, and percents are all the same thing and can be used interchangeably. In other words, ½ = 0.5 = 50%. Those aren't three different numbers; they are just three ways of writing exactly the same number.

Converting from fractions to decimals is easy. Need to know the decimal equivalent of 33/50? Just type 33 ÷ 50 into your calculator, and you'll get 0.66. What about percents? They're easy too. If you look up the word "percent" in the dictionary, you'll see that it really means "per 100." So, 66% really means 66 per 100, or 66/100, or 0.66. If you look at Chevalier's previous math, you'll see why we need to convert back and forth so often—as humans, we like to talk in percents, but we also like to talk about "one chance in six"—so we need a way to convert between these forms. If you are the kind of person who suffers from math anxiety, just relax and practice a few of these on the calculator—you'll have the hang of it in no time.

Rule #2: Zero to One—and That's It!

This one's easy. Probabilities can only range from 0% to 100%, that is, from 0 to 1 (see Rule #1), no less and no more. While you can say there is a 10% chance of something happening, there is no such thing as a –10% chance and certainly no such thing as a 110% chance. A 0% chance of something happening means it won't happen, and a 100% chance means it definitely will. This all might sound obvious, but it points out a major problem with Chevalier's math. Consider his first game with the four dice. He believed that with four dice, he had a 4 × (1/6), or 4/6, or 0.66, or 66% chance of having a six come up. But what if he had seven dice? Then he would have had 7 × (1/6) or 7/6 or 1.17 or 117% chance of winning! And that is certainly wrong—if you roll a die seven times, it might be likely that a six will come up one of those times, but it is not guaranteed (in fact, it is about a 72% chance). Anytime you calculate a probability that comes up greater than 100% (or less than 0%), you know for certain that you've done something wrong.

Rule #3: "Looked For" Divided By "Possible Outcomes" Equals Probability

The first two rules lay some basic groundwork, but now we are going to talk about what probability really is—and it is quite simple. You just take the number of times your "looked for" outcome can come up and divide by the number of possible outcomes (assuming your outcomes are equally likely), and you've got it. What is the chance of a six coming up when you roll a die? Well, there are six possible outcomes, and only one of them is the one we are looking for, so the chance of a six coming up is 1 ÷ 6, or 1/6, or about 17%. What is the chance of an even number coming up when you roll a die? There are 3 even numbers, so the answer is 3/6, or 50%. What is the chance of drawing a face card from a deck of cards? There are twelve face cards in a deck, and fifty-two cards total, so your chances of getting a face card are 12/52, or about 23%. If you understand this, you've got the fundamental idea of probability.

Rule #4: Enumerate!

If Rule #3 is as simple as it sounds (and it is), you might wonder why probability is so tricky. The reason is that the two numbers we need (the number of "looked for" outcomes and the number of possible outcomes) are not always so obvious. For example, if I asked you what the odds of flipping a coin three times and getting "heads" at least twice, what is the number of "looked for" outcomes? I'd be surprised if you could answer that without writing anything down. An easy way to find out the answer is to enumerate all the possible outcomes:

1. HHH
2. HHT
3. HTH
4. HTT
5. THH
6. THT
7. TTH
8. TTT

There are exactly eight possible outcomes. Which ones have heads at least twice? #1, #2, #3, and #5. That's 4 outcomes out of 8 possibilities, so the answer is 4/8, or a 50% chance. Now, why didn't the Chevalier do this with his games? With his first game, there were four die rolls, which means $6 \times 6 \times 6 \times 6$, or 1296 possibilities. It would have been dull work, but he could have enumerated all the possibilities in an hour or so (the list would have looked like 1111, 1112, 1113, 1114, 1115, 1116, 1121, 1122, 1123, etc.), then counted up the number of combinations that had a six in them (671), and divided that by 1296 for his answer. Enumeration will let you solve almost any probability problem, if you have the time. Consider the Chevalier's second game, though: 24 rolls of 2 dice! There are 36 possible outcomes for 2 dice, and so enumerating all 24 rolls would have meant writing down 36^{24} (a number 37 digits long) combinations. Even if he could somehow write down one combination a second, it would have taken longer than the age of the universe to list them all. Enumeration is handy, but when it takes too long, you need to take shortcuts—and that's what the other rules are for.

Rule #5: In Certain Cases, OR Means Add

Very often, we want to determine the chances of "this OR that" happening, such as what are the chances of drawing a face card OR an ace from a deck of cards? When the two things we are talking about are mutually exclusive, that is, when it is impossible for both of them to happen simultaneously, you can add their individual probabilities to get an overall probability. For example, the chances of drawing a face card are 12/52, and the chances of drawing an ace are 4/52. Since these are mutually exclusive events (it is impossible for them both to happen at once), we can add them up: 12/52 + 4/52 = 16/52, or about a 31% chance.

But what if we asked a different question: What are the chances of drawing an ace from a deck of cards or a diamond? If we add these probabilities, we get 4/52 + 13/52 (13 diamonds in a deck) = 17/52. But, if we enumerate, we see this is wrong—the right answer is 16/52. Why? Because the two cases are not mutually exclusive—I could draw the ace of diamonds! Since this case is not mutually exclusive, "or" does not mean add.

Let's look at Chevalier's first game. He seems to be trying to use this rule for his die rolls—adding up four probabilities: 1/6 + 1/6 + 1/6 + 1/6. But he gets the wrong answer, because the four events are not mutually exclusive. The addition rule is handy, but you must be certain the events you are adding up are mutually exclusive from one another.

Rule #6: In Certain Cases, AND Means Multiply

This rule is almost the opposite of the previous one! If we want to find the probability of two things happening simultaneously, we can multiply their probabilities to get the answer—but ONLY if the two events are NOT mutually exclusive! Consider two die rolls. If we want to find the probability of rolling a six on both rolls, we can multiply together the probabilities of the two events: The chance of getting a six on one die roll is 1/6, and also 1/6 for a second die roll. So the chance of getting two sixes is 1/6 × 1/6 = 1/36. You could also have determined that by enumeration, of course, but this is a much speedier way to do it.

In Rule #5, we asked for the probability of drawing an ace OR a diamond from a deck of cards—the rule failed, because the two events were not mutually exclusive. So what if we asked about the probability of drawing an ace AND a diamond? In other words, what is the probability of drawing the ace of diamonds? It should be fairly intuitive that the answer is 1/52, but we can check that with Rule #6, since we know the two events are not mutually exclusive. The chance of getting an ace is 4/52, and the chance of a diamond is 13/52. Multiplying them, 4/52 × 13/52 = 52/2704 = 1/52. So, the rule works and matches our intuition.

Do we have enough rules yet to solve Chevalier's problems? Let's consider his first game:

First Game: In four rolls of a single die, the Chevalier wins if at least one six comes up.

We've already established that we could enumerate this and get the answer 671/1296, but that would take an hour. Is there a quicker way, using the rules we have?

(I'll warn you now—this gets a little hairy. If you don't really care that much, save yourself the headache, and just skip to Rule #7. If you do care, then press on—you will find it worth the effort.)

If the question was about the chances of rolling a die four times and getting four sixes, that would be an AND question for four events that are not mutually exclusive, and we could just use Rule #6: 1/6 × 1/6 × 1/6 × 1/6 = 1/1296. But that isn't what is asked. This is an OR question for four events that are not mutually exclusive (it is possible for the Chevalier to get multiple sixes on the four rolls). So what can we do? Well, one way is to break it down into events that are mutually exclusive and then add them up. Another way to phrase this game is

What are the chances of rolling four dice, and getting either

a. Four sixes, OR
b. Three sixes and one non-six, OR
c. Two sixes and two non-sixes, OR
d. One six and three non-sixes

That might sound a little complicated, but it is four different mutually exclusive events, and if we can figure the probability of each, we can just add them up and get our answer. We've already figured out the probability of (a), using Rule #6: 1/1296. So, how about (b)? Really, (b) is four different mutually exclusive possibilities:

1. 6, 6, 6, non-six
2. 6, 6, non-six, 6
3. 6, non-six, 6, 6,
4. Non-six, 6, 6, 6

The probability of rolling a six is 1/6, the probability of rolling a non-six is 5/6. So, the probability of each of those is 1/6 × 1/6 × 1/6 × 5/6 = 5/1296. Now, if we add up all four, that comes to 20/1296. So, the probability of (b) is 20/1296.

How about (c)? This one is the same as the last, but there are more combinations. It is tricky to figure out how many ways there are for exactly two sixes and two non-sixes to come up, but there are six ways:

1. 6, 6, non-six, non-six
2. 6, non-six, 6, non-six
3. 6, non-six, non-six, 6
4. non-six, 6, 6, non-six
5. non-six, 6, non-six, 6
6. non-six, non-six, 6, 6

And the probability of each of these is 1/6 × 1/6 × 5/6 × 5/6 = 25/1296. Adding up all six of them comes to 150/1296.

This leaves only (d), which is the inverse of (b):

a. Non-six, non-six, non-six, 6
b. Non-six, non-six, 6, non-six
c. Non-six, 6, non-six, non-six
d. 6, non-six, non-six, non-six

The probability of each is 5/6 × 5/6 × 5/6 × 1/6 = 125/1296. Adding up all four gives 500/1296.

So, we have now calculated the probability of the four mutually exclusive events:

a. Four sixes—(1/1296)
b. Three sixes and one non-six—(20/1296)
c. Two sixes and two non-sixes—(150/1296)
d. One six and three non-sixes—(500/1296)

Adding up those four probabilities (as Rule #5 allows) gives us a total of 671/1296, or about 51.77%. So, we can see that this was a good game for the Chevalier—by winning more than 50% of the time, he eventually was likely to make a profit, but the game was close enough to even that his friends believed they had a chance—at least for a while. It certainly is a very different result than the 66% chance of winning the Chevalier believed he had!

This is the same answer we could have gotten from enumeration, but much faster. Really, though, we did a kind of enumeration—it is just that the rules of addition and multiplication let us count everything up much faster. Could we do the same thing to get the answer to Chevalier's second game? We could, but with 24 rolls of two dice, it would probably take an hour or more! This is faster than enumeration, but we can do even better by being tricky—that's where Rule #7 comes in.

Rule #7: One Minus "Does" = "Doesn't"

This is a more intuitive rule. If the chance of something happening is 10%, the chance of it not happening is 90%. Why is this useful? Because often it is quite hard to figure out the chance of something happening but easy to figure out the chance of it NOT happening.

Consider Chevalier's second game. To figure out the chance of double sixes coming up at least once on twenty-four die rolls would be nightmarish to figure out, because you have so many different possible events to add together (1 double sixes, 23 non–double sixes; 2 double sixes, 22 non–double sixes; etc.). On the other hand, what if we ask a different question: What are the chances of rolling two dice twenty-four times and NOT getting double sixes? That is now an AND question, for events that are not mutually exclusive, so we can use Rule #6 to get the answer! But first we'll use Rule #7 twice—watch.

The chance of double sixes coming up on a single roll of the dice is 1/36. So, by Rule #7, the chance of not getting double sixes is 1 – 1/36, or 35/36.

So, using Rule #6 (multiplication), the chances of not getting double sixes 24 times in a row is 35/36 × 35/36 twenty-four times, or as we say $(35/36)^{24}$. You would not want to do this calculation by hand, but using a calculator, you find the answer is around 0.5086, or 50.86%. But that is the chance of the Chevalier losing. To find the chance of the Chevalier winning, we apply Rule #7 again: 1 – 0.5086 = 0.4914, or about 49.14%. Now it is clear why he lost this game! His chances of winning were close enough to even that it was hard for him to tell if this was a winning or losing game, but after playing many times, he was very likely to lose.

Even though all probability problems can be solved through enumeration, Rule #7 can be a really handy shortcut. In fact, we could have used the same rule to solve Chevalier's first game!

Rule #8: The Sum of Multiple Linear Random Selections Is NOT a Linear Random Selection!

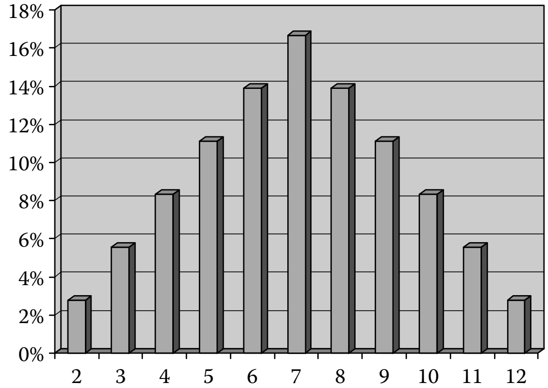
Don't panic. This one sounds hard, but it is really easy. A "linear random selection" is simply a random event where all the outcomes have an equal chance of happening. A die roll is a great example of a linear random selection. If you add up multiple die rolls, though, the possible outcomes do NOT have an equal chance of happening. If you roll two dice, for example, your chance of getting a seven is very good, while your chance of getting a twelve is small. Enumerating all the possibilities shows you why:

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Look at how many 7's there are and only one little twelve! We can show this in a graph, called a probability distribution curve, to visually see the chances of each total coming up:

Rule #8 might seem like a very obvious rule, but I frequently find novice game designers make the mistake of adding together two randomly selected numbers without realizing its effect. Sometimes, it is exactly the effect you want—in the game *Dungeons and Dragons*, players generate (virtual) skill attributes with values ranging from 3 to 18 by rolling three six-sided dice. As a result, you see a lot of attribute values around 10 or 11, but very few at 3 or 18, and this is exactly what the

FIGURE
12.15



designers wanted. How would the game be different if players simply rolled a single twenty-sided die to get their attributes?

Game designers who want to use mechanic of chance as a tool in their games must know what kind of probability distribution curve they want and know how to get it. With practice, probability distribution curves will be a very valuable tool in your toolbox.

Rule #9: Roll the Dice

All the probability we've been talking about so far is **theoretical probability**, that is, mathematically, what *ought to* happen. There is also **practical probability**, which is a measure of what *has* happened. For example, the theoretical probability of getting a 6 when I roll a die is a perfect 1/6, or about 16.67%. I could find the practical probability by rolling a six-sided die 100 times and recording how many times I get a six. I might record 20 sixes out of 100. In that case, my practical probability is 20%, which is not too far from the theoretical probability. Of course, the more trials I do, the closer I would expect the practical probability to get to the theoretical probability. This is sometimes known as the "Monte Carlo" method, after the famous casino.

The great thing about the Monte Carlo method of determining probability is that it doesn't involve any complex math—you just repeat the test over and over again and record how it comes out. It can sometimes give more useful results than theoretical probability too, because it is a measure of the real thing. If there is some factor that your mathematics didn't capture (e.g., perhaps your die is slightly weighted toward sixes), or if the math is just so complicated that you can't come up with a theoretical representation of your case, the Monte Carlo method can be just the thing. The Chevalier could easily have found good answers to his questions by just rolling the dice again and again, counting up wins, and dividing by the number of trials.

And here in the computer age, if you know how to do a little bit of programming (or know someone who can—see Rule #10), you can easily simulate millions of trials in just a few minutes. It isn't too hard to program simulations of games and get some very useful probability answers. For example, in Monopoly, which squares are landed on most frequently? It would be nearly impossible to figure this out theoretically—but a simple Monte Carlo simulation allows you to answer the question quickly by using a computer to roll the dice and move the pieces around the board a few million times. Alternatively, you could make use of the *Machinations* system created by Joris Dormans, which is specifically designed to model gameplay systems and show patterns of results through repeated simulations.

Rule #10: Geeks Love Showing Off (Gombaud's Law)

This is the most important of all the probability rules. If you forget all the others but remember this one, you'll get by just fine. There are many more difficult aspects of probability that we won't get into here—when you run into them, the easiest thing to do is to find someone who considers themselves a "math whiz." Generally, these people are thrilled to have someone actually needing their expertise, and they will bend over backwards to help you. I have used Rule #10 to solve hard game design probability questions again and again. If there aren't any experts around you, post your question on a forum or mailing list. If you really want a fast response, preface it with "This problem is probably too difficult for anyone to solve, but I thought I would ask anyway," for there are many math experts who love the ego boost of solving a problem that others think is impossible. In a sense, your hard problem is a game for them—why not use game design techniques to make it as attractive as possible?

You might even be doing your geek a favor! I like to call Rule #10 "Gombaud's Law," in honor of Antoine Gombaud, the Chevalier de Méré, who, through his awareness of this principle, not only solved his gambling problem (his mathematical one, anyway), but inadvertently initiated all of probability theory.

You might be afraid of exercising Rule #10, because you are afraid of asking stupid questions. If you feel that way, don't forget that Pascal and Fermat owed the Chevalier a great debt—without his stupid questions, they never would have made some of their greatest discoveries. Your stupid question might lead to a great truth of its own—but you'll never know unless you ask.

Expected Value

You will use probability in many ways in your designs, but one of the most useful will be to calculate **expected value**. Very often, when you take an action in a game, the action will have a value, either positive or negative. This might be points, tokens, or money gained or lost. The expected value of a transaction in a game is the average of all the possible values that could result.

For example, there might be a rule in a board game that when a player lands on a green space, he can roll a six-sided die and get that many power points. The expected value of this event is the average of all the possible outcomes. To get the average in this case, since all the probabilities are equal, we can add up all the possible die rolls, 1 + 2 + 3 + 4 + 5 + 6 = 21, and divide by 6, which gives us 3.5. As a game designer, it is very useful for you to know that each time someone lands on a green space, they will, on average, get 3.5 power points.

But not all examples are so simple—some involve negative outcomes, and outcomes that aren't evenly weighted. Consider a game where a player rolls two dice. If they get a 7's, or an 11's, they win $5, but if they get anything else, they lose $1. How do we figure out the expected value of this game?

The chance of rolling a 7 is 6/36.

The chance of rolling an 11 is 2/36.

Using Rule #8, the chance of rolling anything else is 1 − 8/36, or 28/36.

So, to calculate the expected value, we multiply the probabilities by the values for each and add them all up, like this:

Outcome	**Chance×Outcome**	**Value**
7	6/36×$5	$0.83
11	2/36×$5	$0.28
Everything else	28/36×− $1	−$0.78
Expected value		$0.33

So, we see that this is a good game to play, because in the long run, you will, on average, win thirty-three cents each time you play. But, what if we changed the game, so that only 7's are winning numbers and 11's make you lose a dollar, just like all the other numbers? This changes the expected value, like this:

Outcome	**Chance×Outcome**	**Value**
7	6/36×$5	$0.83
Everything else	30/36×− $1	−$0.83
Expected value		$0.00

An expected value of zero means that this game is just as good as flipping a coin in the long run. Wins and losses are completely balanced. What if we change it again, so that this time only eleven wins?

Outcome	**Chance×Outcome**	**Value**
11	2/36×$5	$0.28
Everything else	34/36×− $1	−$0.94
Expected value		−$0.86

Ouch! As you might expect, this is a losing game. You'll lose, on average, about eighty-six cents each time you play it. Of course, you could make it into a fair game, or even a winning game, by increasing the payoff for getting an eleven.

Consider Values Carefully

Expected value is an excellent tool for game balancing, which we will discuss more in the next chapter—but if you aren't careful about what the true value of an outcome is, it can be very misleading.

Consider these three attacks that might be part of a fantasy role-playing game:

Attack Name	Chance of Hitting (%)	Damage
Wind	100	4
Fireball	80	5
Lightning bolt	20	40

What is the expected value of each of these? Wind is easy—it always does exactly 4 damage, so the expected value of that attack is 4. Fireball hits 80% of the time and misses 20% of the time, so it's expected value is $(5 \times 0.8) + (0 \times 0.2) = 4$ points, the same as the wind attack. The lightning bolt attack doesn't hit very often, but when it does, it packs a wallop. Its expected value is $(40 \times 0.2) + (0 \times 0.8) = 8$ points.

Now, based on those values, one might conclude that players would always use the lightning bolt attack, since on average it does double the damage of the other two attacks. And if you are fighting an enemy that has 500 hit points, that might be correct. But what about an enemy with 15 hit points? Most players would not use lightning bolt in that case—they would opt for something weaker but surer. Why is this? Because even though the lightning bolt can do 40 damage points, only 15 of them are of any use in that situation—the real expected value of the lightning bolt against an enemy with 15 HP is $(0.2 \times 15) + (0.8 \times 0) = 3$ points, which is lower than both the wind and the fireball attack.

You must always take care to measure the real values of actions in your game. If something gives a benefit that a player can't use, or contains a hidden penalty, you must capture that in your calculations.

Human Element

You must also keep in mind that expected value calculations do not perfectly predict human behavior. You would expect players to always choose the option with the highest expected value, but that is not always the case. In some cases, this is due to ignorance—because players did not realize the actual expected value. For example, if you didn't tell players the respective chances of wind, fireball, and lightning bolt, but left it to them to discover them through trial and error, you might find that players who tried lightning bolt several times and never got a hit reached the conclusion that "lightning bolt never hits" and therefore has an expected value of zero. The estimates that players make about how often an event happens are often incorrect. You must be aware of the "perceived probabilities" that players have arrived at, because it will determine how they play.

But sometimes, even with perfect information, players still will not choose an option with the highest expected value. Two psychologists, Kahneman and Tversky, tried an interesting experiment, where they asked a number of subjects which of the two games they would like to play:

Game A:

66% chance of winning $2400

33% chance of winning $2500

1% chance of winning $0

Game B:

100% chance of winning $2400

These are both pretty great games to play! But is one better than the other? If you do the expected value calculations

$$\text{Expected Value of Game A: } 0.66 \times \$2400 + 0.33 \times \$2500 + 0.01 \times \$0 = \$2409$$

$$\text{Expected Value of Game B: } 1.00 \times 2400 = \$2400$$

You can see that Game A has a higher expected value. But only 18% of the subjects they surveyed picked A, while 82% preferred playing Game B.

Why? The reason is that the expected value calculation does not capture an important human element: regret. People not only seek out options that create the most pleasure, they also avoid the ones that cause the most pain. If you played Game A (and we're assuming you only get to play it once), and were unlucky enough to get that 1% and $0, it would feel pretty bad. People are often willing to pay a price to eliminate the potential of regret—"buying peace of mind," as the insurance salesmen say. Not only are they willing to pay a price to avoid regret, they are willing to take risks. This is why a gambler who has lost a little money is often willing to take more risks to try to get the money back. Tversky puts it this way: "When it comes to taking risks for gains, people are conservative. They will make a sure gain over a problem gain. But we are also finding that when people are faced with a choice between a small, certain loss and a large, probable loss, they will gamble." This appears to be a large part of the success of the "free-to-play" game *Puzzle & Dragons*. Players perform a series of puzzles and rack up treasures while making their way through a dungeon. Sometimes, though, they perish in the dungeon, and the game effectively says, "Oh, that's too bad, you're dying. Look at all the treasure you are going to lose. Are you sure you don't want to pay just a little bit of real money, so you have a shot at keeping what you have earned?" And many people respond by paying cash money to avoid that small, certain loss.

In some cases, the human mind inflates some risks completely out of proportion. In one study, Tversky asked people to estimate the likelihood of various causes of death and obtained the following results:

Cause of Death	Estimated Chance (%)	Actual Chance (%)
Heart disease	22	34
Cancer	18	23
Other natural causes	33	35
Accident	32	5
Homicide	10	1
Other unnatural causes	11	2

What is particularly interesting here is that the subjects making estimates *under-estimated* the top three categories (natural causes of death) and significantly *over-estimated* the bottom three (unnatural causes of death). This distortion of reality seems to be a reflection of the fears of the respondents. What bearing does this have on game design? As a designer, you must have not only a grasp of the actual probabilities of events in your game but also the *perceived* probabilities, which may be quite different for a number of reasons.

You will need to consider both actual and perceived probabilities when calculating expected values, which provide such useful information that they make Lens #35.

Lens #35: The Lens of Expected Value

To use this lens, think about the chance of different events occurring in your game and what those mean to your player.

Ask yourself these questions:

- What is the actual chance of a certain event occurring?
- What is the perceived chance?
- What value does the outcome of that event have? Can the value be quantified? Are there intangible aspects of value that I am not considering?
- Each action a player can take has a different expected value when I add up all the possible outcomes. Am I happy with these values? Do they give the player interesting choices? Are they too rewarding, or too punishing?

Expected value is one of your most valuable tools for analyzing game balance. The challenge of using it is finding a way to numerically represent everything that can happen to a player. Gaining and losing money is easy to represent. But what is the numerical value of "boots of speed" that let you run faster or a "warp gate" that lets you skip two levels? These are difficult to quantify perfectly—but that doesn't mean you can't take a guess. As we'll see in the next chapter, as you go through multiple iterations of game testing and tweaking parameters and values in your game, you will also be tweaking your own estimations of the values of different outcomes. Quantifying these less tangible elements can be quite enlightening, because it makes you think concretely about what is valuable to the player and why—and this concrete knowledge will put you in control of the balance of your game.

Skill and Chance Get Tangled

As tricky as probability and the difference between actual and perceived values might be, the game mechanic of chance has more tricks up its sleeve. As much as we like to think that chance and skill are completely separate mechanics, there are important interactions between them that we cannot ignore. Here are five of the most important skill/chance interactions for a game designer to consider.

1. **Estimating chance is a skill**: In many games, what separates the skilled players from the unskilled is their ability to predict what is going to happen next, often through calculating probabilities. The game of blackjack, for example, is almost entirely about knowing the odds. Some players even practice "card counting," which is the practice of keeping track of what cards have already been played, since each card played changes the odds of what subsequent cards can appear. The perceived probabilities in your game can vary a great deal between players who are skilled estimators and those who are not.
2. **Skills have a probability of success**: Naively, one might think that completely skill-based games, such as chess or baseball, have no aspects of randomness or risk in them. But from a player's point of view, this simply isn't true. Every action has some level of risk, and players are constantly making expected value decisions, deciding when to play it safe and when to take a big risk. These risks can be difficult to quantify (what are the odds that I can successfully steal a base or that I can trap my opponent's queen without him noticing?), but they are still risks. When designing a game, you need to make sure they are balanced just as you would balance "pure chance" game elements, like drawn cards or die rolls.
3. **Estimating an opponent's skill is a skill**: A big part of a player's ability to determine the chances of success for a particular action rests on their ability to estimate their opponent's skill. A fascinating part of many games is trying to fool your opponent into thinking your skills are greater than they are, to prevent him from trying

anything too bold and to make him uncertain of himself. Likewise, sometimes the opposite is true—in some games, it is a good strategy to make a player think your skills are less than they really are, so that your opponent will not notice your subtle strategies and will perhaps try actions that would be risky against a skilled player.

4. **Predicting pure chance is an imagined skill**: Humans look for patterns, consciously and subconsciously, to help predict what is going to happen next. Our mania for patterns often leads us to look for and find patterns where none exist. Two of the most common false patterns are the "lucky streak fallacy" (I've had several wins in a row, and therefore another is likely) and, its opposite, the "gambler's fallacy" (I've had several losses, so I must be due for a win). It is easy to scoff at these as ignorant, but in the all-important mind of the player, detecting these bogus patterns feels like the exercise of a real skill, and as a designer, you should find ways to use that to your advantage.
5. **Controlling pure chance is an imagined skill**: Not only do our brains actively seek patterns, but they also actively and desperately seek cause-and-effect relationships. With pure chance, there is no way to control the outcome—but that doesn't stop people from rolling the dice a certain way, carrying lucky charms, or engaging in other superstitious rituals. This feeling that it might be possible to control fate is part of what makes gambling games so exciting. Intellectually, we know it isn't possible, but when you are up there rolling the dice, saying "come on, come on…," it certainly feels like it might be possible, especially when you get lucky! If you try playing games of pure chance, but completely disengage yourself from the idea that anything you think or do can influence the outcome, much of the fun suddenly drains away. Our natural tendency to try to control fate can make games of chance feel like games of skill.

Chance is tricky stuff, because it intertwines hard math, human psychology, and all of the basic game mechanics. But this trickiness is what gives games their richness, complexity, and depth. The last of our seven basic game mechanics gives us Lens #36.

Lens #36: The Lens of Chance

To use this lens, focus on the parts of your game that involve randomness and risk, keeping in mind that those two things are not the same.

Ask yourself these questions:

- What in my game is truly random? What parts just feel random?
- Does the randomness give the players positive feelings of excitement and challenge, or does it give them negative feelings of hopelessness and lack of control?

- Would changing my probability distribution curves improve my game?
- Do players have the opportunity to take interesting risks in the game?
- What is the relationship between chance and skill in my game? Are there ways I can make random elements feel more like the exercise of a skill? Are there ways I can make exercising skills feel more like risk taking?

Risk and randomness are like spices. A game without any hint of them can be completely bland, but put in too much and they overwhelm everything else. But get them just right, and they bring out the flavor of everything else in your game. Unfortunately, using them in your game is not as simple as sprinkling them on top. You must look into your game to see where elements of risk and randomness naturally arise, and then decide how you can best tame them to do your bidding. Don't fall into the trap of thinking that elements of chance only occur around die rolls or randomly generated numbers. On the contrary, you can find them wherever a player encounters the unknown.

At long last, we have made it through all seven of the basic game mechanics. Soon, we will move onto more advanced mechanics that are built from these, such as puzzles and interactive story structures. But first, we need to explore methods of bringing these basic elements into balance.

Other Reading to Consider

***Game Mechanics*: *Advanced Game Design* by Ernest Adams and Joris Dormans**. This book gets into a lot of wonderfully nitty-gritty details about the interactions of various game mechanics and gives an introduction to the fascinating *Machinations* system for simulating your game design.

***The Oxford Book of Board Games* by David Parlett**. Contains more details on Parlett's Rule Analysis, as well as descriptions of some amazing but little-known board games from previous centuries.

***Uncertainty in Games* by Greg Costikyan**. An incredibly insightful book about the nature of chance and uncertainty in games. I get something new from it every time I read it.

***The Unfinished Game*: *Pascal, Fermat, and the Unfinished Letter that Made the World Modern* by Keith Devlin**. If you want even more details of the story of how probability came to be, this is the definitive book.